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Black Holes and Strings: the Polymer Link

Ramzi R. Khuri*

Department of Physics
Queen Mary and Westfield College
Mile End Road
London E1 4NS UK

Abstract

Quantum aspects of black holes represent an important testing ground for a theory of quantum gravity. The recent success of string theory in reproducing the Bekenstein-Hawking black hole entropy formula provides a link between general relativity and quantum mechanics via thermodynamics and statistical mechanics. Here we speculate on the existence of new and unexpected links between black holes and polymers and other soft-matter systems.

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The standard model of elementary particle physics has proven very succesful in describing three of the four fundamental forces of nature. In the most optimistic scenario, the standard model can be generalized to take the form of a grand unified theory, in which quantum chromodynamics, describing the strong force, and the electroweak theory, unifying the weak interaction with electromagnetism, are synthesized into a single theory in which all three forces have a common origin. The underlying framework of particle physics is quantum mechanics, in which the natural length scale associated with a particle of mass m (such as an elementary particle) is given by the Compton wavelength $\lambda = \hbar/mc$, where $\hbar$ is Planck's constant divided by 2π and c is the speed of light. Scales less than λ are therefore unobservable within the context of the quantum mechanics of this particle.

Quantum mechanics, however, has so far proven unsuccessful in describing the fourth fundamental force, gravitation. The successful theory in this case is that of general relativity, which, however, does not lend itself to a straightforward attempt at quantization. The main problem in such an endeavour is that the divergences associated with trying to quantize gravity cannot be circumvented (or "renormalized") as they are for the strong, weak and electromagnetic forces.

Among the most interesting objects predicted by general relativity are black holes, which represent the endpoint of gravitational collapse. According to relativity, an object of mass m under the influence of only the gravitational force (*i.e.,* neutral with respect to the other three forces) will collapse into a region of spacetime bounded by a surface, the event horizon, beyond which signals cannot be transmitted to an outside observer. The event horizon for the simplest case of a static, spherically symmetric black hole of mass m is located at a radius $R = 2Gm/c^2$, the Schwarzschild radius, from the collapsed matter at the center of the sphere, where G is Newton's constant.

In trying to reconcile general relativity and quantum mechanics, a natural question to ask is whether they have a common domain. This would arise when an elementary particle exhibits features associated with gravitation, such as an event horizon. This may occur

provided $\lambda \lesssim R$, which implies that, even within the framework of quantum mechanics, an event horizon for an elementary particle may be observable. Such a condition is equivalent to $m \gtrsim m_P = \sqrt{\hbar c/G} \sim 10^{19} GeV$, the Planck mass, or $\lambda \lesssim l_P = \sqrt{\hbar G/c^3}$, the Planck scale. It is in this domain that one may study a theory that combines quantum mechanics and gravity, the so-called *quantum gravity* (henceforth we use units in which $\hbar = c = 1$).

A problem, however, arises in this comparison, because most black holes are thermal objects, and hence cannot reasonably be identified with pure quantum states such as elementary particles. In fact, in accordance with the laws of *black hole thermodynamics* [1], black holes radiate with a (Hawking) temperature constant over the event horizon and proportional to the surface gravity: $T_H \sim \kappa$. Furthermore, black holes possess an entropy $S = A/4G$, where A is the area of the horizon (the area law), and $\delta A \geq 0$ in black hole processes. So only a black hole with zero area can correspond to a pure state with $S = 0$ such as an elementary particle, while a black hole with nonzero area, and therefore nonzero entropy, corresponds to an *ensemble* of states. A question, then, that can be posed of a theory of quantum gravity is the following: since the basis of ordinary thermodynamics is (quantum) statistical mechanics, can one recover the laws of black hole thermodynamics by the counting of microscopic states? In particular, can one recover the area law from a quantum mechanical entropy arising as the logarithm of the degeneracy of quantum states?

At the present time, string theory, the theory of one-dimensional extended objects, is the only known reasonable candidate theory of quantum gravity. The divergences inherent in trying to quantize point-like gravity seem not to arise in string theory. Furthermore, string theory has the potential to unify all four fundamental forces within a common framework. At an intuitive level, one can see how point-like divergences may be avoided in string theory by considering scattering amplitudes in string theory [2]. Unlike those of field theory, the four-point amplitudes in string theory do not have well-defined vertices at which the interaction can be said to take place, hence no corresponding divergences associated with the zero size of a particle. A simpler way of saying this is that the finite

size of the string smooths out the divergence of the point particle.

For the purpose of understanding black hole thermodynamics, an important feature of string theory is that classical solutions [3] may be easily constructed as composites of single-charged fundamental constituents. Identifying these constituents with states in string theory, one can compare the Bekenstein-Hawking entropy obtained from the area of the classical solution to the quantum-mechanical microcanonical counting of ensembles of states [4]. For example, the extremal Reissner-Nordström charged black hole solution of Einstein-Maxwell theory arises in string theory as the composite of four charges, N_1, N_2, N_3 and N_4, normalized to correspond to number operators in string theory. The area law then yields a Bekenstein-Hawking entropy $S_{BH} = 2\pi\sqrt{N_1 N_2 N_3 N_4}$. The counting of the degeneracy of the states forming this black hole leads to the same quantity $S_{QM} = \ln d(N_i) = S_{BH}$. Even in the black hole picture, this result can be seen to arise from the number of ways in which the various constituents combine. Following [5], one can write four-centered solutions each with charge N_i of a given species. A black hole with nonzero area is formed when all charges are brought together to the same point. The precise partition function [6] yielding the correct degeneracy $d(N_i) = exp(S_{BH})$ is obtained provided both bosonic and fermionic excitations of a supersymmetric string-like object along various dimensions are taken into account.

The recovery of the area law in a wide variety of contexts in string theory suggests that we have accounted for the microscopic degrees of freedom of the black hole. However, the ensemble of string states on the one hand and the black hole on the other represent two very different objects, so we must try to understand the correspondence between them [7]. For simplicity, let us consider the case of a long, self-gravitating string in $D = 4$ dimensions [8]. At level N, a free string has mass $M \sim \sqrt{N}/l_s$, size $L \sim N^{1/4} l_s$ and entropy $S \sim \sqrt{N}$, where l_s is the string scale. This picture is valid provided the string coupling $g << 1$, where g is related to Newton's constant G via $G \sim g^2 l_s^2$. This picture represents a random walk [9] with $n = \sqrt{N}$ steps, each a single string "bit" of length l_s [10].

Let us now slowly increase the coupling g. As shown in [8], gravitational effects start becoming strong at $g_0 \sim N^{-3/8} = n^{-3/4}$, after which the string collapses until it reachers the size of the string scale l_s. At the critical coupling $g_c \sim N^{-1/4} = n^{-1/2}$, the Schwarzschild radius $R = 2GM$ of a black hole with the same mass becomes of the order of the string scale, and one can sensibly start thinking of the string as a black hole. At this point, too, the entropies match: $S_{BH} \sim R^2/G = 1/g_c^2 = \sqrt{N} = n$. For $g > g_c$, the black hole picture prevails. In the intermediate range $g_0 < g < g_c$, the size of the string state was shown using a thermal scalar field theory in [8] to be

$$L \sim \frac{l_s}{g^2 N^{1/2}} = \frac{l_s}{g^2 n}, \tag{1}$$

which smoothly interpolates between the random walk size and the string scale. Note that for n large, the coupling is small throughout the ranges we are considering. This is an interesting result with a specific prediction for the coupling dependence of the size of the string as it collapses into a black hole. A natural question to ask is whether this sort of result also arises in analogous physical systems already considered. Since random walks with interactions arise in polymer physics [11,10], the relation (1) should also hold for a self-attracting polymer chain.

We start with a random walk with n steps each of size a, so that the size of the polymer is initially given by $L_0 = \sqrt{n}a$. Suppose we place the polymer in a medium of scatterers of number density ρ and (dimensionless) potential strength u. Then the size of the polymer was shown to be [12]

$$L^2 = x^{-2}\left(1 - \exp(-nx^2a^2)\right), \tag{2}$$

where $x = u\rho a^2$ can be thought of as an effective scattering cross section.

To compare with a self-gravitating string with $a = l_s$, the scatterers are taken to coincide with the positions of the string bits themselves. For large n and in a mean-field approximation, the number density of n bits in a volume L_0^3 is given by

$$\rho = \frac{n}{(n^{3/2} l_s^3)} = n^{-1/2} l_s^{-3}. \tag{3}$$

For g small, the leading order interaction potential is given by

$$\frac{u}{l_s} \sim \sum_{i,j} \frac{g^2}{|\vec{r}_i - \vec{r}_j|} \sim \frac{g^2 n^2}{L_0} = \frac{g^2 n^{3/2}}{l_s}, \tag{4}$$

where $\vec{r}_i$ is the position of the ith link. It follows that $x \sim ng^2/l_s$, so that

$$L^2 = l_s^2 n^{-2} g^{-4} \left(1 - \exp(-n^3 g^4)\right). \tag{5}$$

For $g < g_0 = n^{-3/4}$, $L^2 \simeq n l_s^2$ which is the random walk, corresponding to the free string. As in the string case, a transition occurs at $g \sim g_0$. As g is increased past g_0, the size quickly shrinks to $L^2 \simeq l_s^2/n^2 g^4 = l_s^2/g^2 N$, as in (1). This kind of relation holds[1] until $g \sim g_c \sim n^{-1/2}$, when $L \sim R$, the Schwarzschild radius of the polymer, and the black hole picture dominates.

This connection between black holes, strings and polymers is very interesting and merits further investigation. Similar links with other soft-matter systems have also been noted in [13], where the area law was recovered for the case of a liquid field theory and where it was argued that the area law contributions to the free energy are primarily responsible for liquid surface tension. The speculation was also made that the area law arises in the context of protein folding.

Connections between physical and biological systems are always exciting. The cases discussed above are especially so since quantum gravity is generally considered too remote to have relevance to other areas of physics, much less other fields of science. In particular, the fascinating possibility arises that mathematical techniques used to study black holes can be useful in understanding biological questions, such as protein dynamics, while methods of polymers physics can potentially shed light on quantum gravity.

[1] Once the self-interaction of the polymer becomes strong, the simple result (5) is no longer exact and a more precise computation is required. Nevertheless, it is clear that one obtains a smooth transition from the random walk to the Schwarzschild radius via a nonperturbative coupling dependence, so that even if (1) is not exactly recovered, it remains a good approximation for the collapse of the polymer.

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